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Exploring How Digital and Physical Tools Differently Influence Preservice Teachers' Understanding

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Abstract

This study examines the effectiveness of using a digital tool (GeoGebra) and a physical tool (tracing paper) to support preservice teachers' understanding of rotation. A quasi-experimental, pretest–posttest design was conducted with 17 preservice teachers, divided into two class groups – one using a digital tool and the other a physical tool intervention – at a state university in the United States. A content assessment was administered to all participants both before and after their participation in one of the interventions. Results from the Wilcoxon Signed-Rank test indicate significant improvement in preservice teachers' relevant content knowledge after using either tool. No significant difference was found between the two intervention groups, suggesting that both digital and physical tools can provide comparable positive effects on key aspects of rotation. Further descriptive statistics revealed that these two tools helped learners understand rotation in varied yet complementary aspects. Therefore, it is promising to integrate both into instruction for more effective learning outcomes.

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Introduction

Transformational geometry lays a crucial foundation for understanding the concepts of congruence and similarity at the middle school level (NGA Center & CCSSO, 2010). Rigid transformations – including translations, reflections, and rotations – are defined as changes in position without changes in the size or shape of objects (Van de Walle et al., 2019). Research indicates that rotations are especially challenging for students to understand compared to the other two types of transformation, which are more cognitively intuitive (Duval, 1999). These challenges include identifying the center of rotation, discerning the angle and direction of rotation, and preserving the shape's congruence.

While students often struggle with basic geometric concepts and problem solving, extensive research attributes these difficulties to inadequacies in teachers' content knowledge and instructional methods (Ball et al., 2008; Mbusi & Luneta, 2021; Mukuka & Alex, 2024). Teachers who do not know a subject well tend to be less confident in helping students learn this content (Ball et al., 2008). As a result, these teachers often avoid or reduce instructional time on challenging topics such as rotations and proofs (Schoenfeld, 2004), focusing instead on mechanical memorization of formulas, theorems, and properties (Clements & Battista, 1992). They often follow the textbook without adapting activities or materials based on students' needs (Ball et al., 2008). Due to their limited ability to provide guidance or ask probing questions, these teachers may discourage student exploration, discussion, or explanation (Hill et al., 2008). In addition, such teachers are less likely to use digital or physical tools to enhance students' geometric visualization and reasoning (Battista, 2007; Ng & Sinclair, 2015). Therefore, efforts to empower teachers with stronger geometric knowledge and skills are essential for effective teacher preparation (Mbusi & Luneta, 2021).

One such effort has focused on integrating digital tools (which can also be referred to as technological tools) or physical tools (manipulatives) into instruction to support the exploration of geometric concepts and relationships (Van de Walle et al., 2019). For decades, digital tools such as GeoGebra, Desmos, Geometry Pad, and Sketchpad have become increasingly popular in classrooms (Ng & Sinclair, 2015; Ndungo et al., 2025). A wealth of studies has documented the advantages of digital tools, including immediate visual feedback, dynamic visualization, multiple representations that connect algebraic and geometric ideas, increased accuracy, and enhanced motivation and engagement (Hohenwarter & Jones, 2007; Zbiek et al., 2007; Hoyles & Noss, 2003; Kaput, 1992; NCTM, 2000; Laborde, 2001). Physical tools, on the other hand, remain valuable due to features such as the concrete representation of geometric objects, tactile experiences that help understand abstract concepts, increased interest, and support for the development of coordination and spatial reasoning (Clements & Battista, 1992; Moyer & Jones, 2004; Olkun, 2003).

Despite mounting research into the advantages of both digital and physical tools for teaching and learning across various mathematical topics, little has been done to compare their effectiveness specifically in teaching the concept of rotation – a topic that remains challenging for both teachers and students. Moreover, limited research has explored the possibility of integrating both physical and digital tools to help teachers and students grasp geometric ideas, such as rigid transformations, by bridging concrete and abstract representations.

Statement of Problem and Research Questions

Given all the points discussed above, there is a pressing need to assess the effectiveness of digital and physical tools on teachers' learning of transformations, particularly the topic of rotation, as this has a significant impact on student understanding. This concern can be addressed through a study involving two interventions: the use of a digital tool (GeoGebra) with one group of preservice teachers and a physical tool (tracing paper) with another group. Teachers' understanding of rotations and their principal components — the center of rotation, angle, and direction — will be evaluated before and after the interventions. This study will contribute to the body of research on rotations and provide insights into the potential for integrating both physical and digital tools in instruction, rather than relying exclusively on one approach over the other. The study sought to answer the following research questions.

1) What foundational knowledge do preservice teachers possess about rotation and its key components?
2) Does the use of digital or physical tools improve preservice teachers' understanding of rotation, and if so, in which aspects?
3) How do the two interventions differ in their effects on key aspects of preservice teachers' understanding of rotation?

Literature Review

Theoretical Perspectives on Preservice Teachers' Knowledge for Teaching

The long-standing concern regarding students' underperformance in geometry calls for interventions in geometry education—not only in K–12 classrooms but also with teacher preparation programs (Duval, 1999; Mbusi & Luneta, 2021; Mukuka & Alex, 2024). It is important to determine which types of teacher knowledge require interventions and to identify their underlying causes. Shulman (1987) outlined a knowledge base with seven categories that are essential for teachers to promote comprehension among students. These categories comprise content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, and knowledge of educational ends, purposes, and values. Building on Shulman's work, Ball et al. (2008) proposed six domains of teacher knowledge, including common content knowledge and five others that require varying levels of teaching experience and understanding of students—such as specialized content knowledge, knowledge of content and teaching, and knowledge of content and students. Common content knowledge is defined as the mathematical knowledge and skills shared with others who know and use mathematics across varied areas or fields (Ball et al., 2008).

Common content knowledge is considered essential, as studies have found that teachers who lack it face difficulties and feel less confident in their teaching (Ball et al., 2008; Shing et al., 2015). Veal and MaKinster (1999) suggest that strong content knowledge is vital for the development of pedagogical content knowledge, which is central to the knowledge needed for teaching. This is supported by Ma's (1999) comparative study of 23 American and 72 Chinese elementary mathematics teachers. Ma found that Chinese teachers' profound understanding of fundamental mathematics (PUFM) enabled them to present concepts flexibly from multiple perspectives and to link ideas in ways that help students develop deep, connected understandings, rather than merely learning procedures. Based on these findings, Ma (1999) suggests the teacher preparation should focus on fostering prospective teachers' deep, connected understanding of content knowledge through reasoning and integration of content and pedagogy across grade levels, thereby promoting a holistic view of math curriculum.

In terms of mathematical content knowledge, research has consistently revealed that geometry is one area in which preservice teachers struggle the most. For instance, Reinke (2010) found that many preservice teachers relied excessively on memorized formulas and, therefore, calculated perimeters and areas without fully understanding their distinct attributes. Through a large-scale study involving 757 preservice teachers in Ghana, Akayuure (2021) reported that participants demonstrated stronger procedural knowledge (sketching or calculation) and declarative knowledge (facts and attributes) than conditional knowledge (geometrical reasoning). The study also revealed that tasks involving three-dimensional figures (prisms and pyramids) were more challenging than two-dimensional figures (angles, triangles, and quadrilaterals) to most preservice teachers. Additionally, through a survey of 103 preservice teachers in Ghana, Salifu et al. (2020) identified a few reasons contributing to their difficulties in learning geometry, including incomplete coverage of geometry curriculum, insufficient study after class, lack of real-life applications of geometry, limited interest, and inadequate problem-solving practice. Other studies identified additional factors such as weak prior knowledge, underdeveloped spatial visualization and reasoning skills, limited comprehension of geometrical terminology, and inadequate access to teaching resources (Mason, 2002; Uduosoro, 2011; Aysen, 2012).

Drawing on the findings of prior research, our study focuses on preservice teachers' content knowledge of rotations for several reasons. First, the fundamental importance of subject content knowledge has been consistently emphasized, as teachers must be able to perform the mathematical tasks that they assign to their students (Mukuka & Alex, 2024). Second, relatively few research studies have investigated the geometric content knowledge of preservice teachers, particularly with respect to rotations and their properties ((Mbusi, 2021). Third, there is a pressing need to investigate interventions aimed at improving teachers' mathematical content knowledge in order to enhance student learning (Mukuka & Alex, 2024).

Research Studies on Digital and Physical Tools in Mathematics Education

It is important to note that a tool does not illustrate a mathematical concept or impart knowledge on its own (Thompson, 1994; Van de Walle, 2019). Rather, the tool serves to visualize a concept through representation while the learner's mind imposes mathematical relationships and connections. Therefore, selecting appropriate tools or using them effectively can lead to desired learning outcomes. Otherwise, they may cause limited understanding or misconceptions.

Given the growing emphasis on integrating digital tools into math education, it is crucial to clarify their scopes and assess their effectiveness. According to Moyer-Packenham and Westenskow (2013), digital tools encompass at least three broad categories: virtual manipulatives (VMs) (e.g., virtual representations of physical manipulatives available on the National Library of Virtual Manipulatives or other online platforms), dynamic software programs (e.g., Geometer's Sketchpad and GeoGebra), and other online learning resources (such as instructional videos and practice exercises). Through a meta-analysis of 66 studies examining the effects of virtual manipulatives (VMs) on student achievement, Moyer-Packenham and Westenskow (2013) reported a moderate effect of VM compared with other instructional treatments, including both physical manipulatives and textbook instruction. Further, this synthesized study identified five key affordances of VMs: (1) enabling focused attention, (2) simultaneously linking multiple representations with students' actions, (3) cultivating creativity and multiple solutions, (4) supporting precise representations and accuracy, and (5) increasing motivation and engagement. Similar benefits have been reported for other types of digital tools (Hohenwarter & Jones, 2007; Hoyles & Noss, 2003; Ndungo et al., 2025; Zbiek et al., 2007).

On the other hand, the traditional physical manipulatives have long played an important role in mathematics education. Through examining the instructional practice of 10 middle grades teachers, Moyer and Jones (2004) found that 70% of their lessons incorporated a mathematical tool like physical manipulative, calculator, or measurement device. Research suggests that students who use manipulatives tend to outperform their peers who do not (Suydam, 1986). In a meta-analysis of 55 studies involving concrete manipulatives, researchers reported a small to moderate overall positive effective size in favor of using manipulatives over mathematics instruction relying solely on abstract symbols (Carbonneau et al., 2013). The study also revealed a strong effect on retention, but only small effects on problem solving, transfer, and justification. The authors suggest: (1) providing proper guidance when using manipulatives, (2) offering manipulatives to students in earlier developmental stages or those who struggle, (3) using physical manipulatives to support the development of conceptual understanding.

In summary, despite extensive studies on the application of digital and physical tools in K-12 classrooms and their impact on student performance, few reports address how preservice teachers benefit from these tools in their mathematical learning – learning that ultimately strengthens their common content knowledge (or subject content knowledge) and teaching skills. This study aims to fill that gap by evaluating and comparing preservice teachers' performance using a digital tool (GeoGebra) and a physical tool (tracing paper) in a geometric topic.

Method

Research Design

This study employed a quasi-experimental, pretest-posttest design with non-equivalent groups (Reichardt, 2009) to compare the effectiveness of interventions of digital and physical tools in enhancing preservice teachers' conceptual understanding of rotations in a mathematics content course. By assessing preservice teachers' relevant content knowledge before and after the interventions, the study examined their knowledge gains and potential contributing factors. In addition, this study aimed to identify possible disparities in terms of the centers of rotation, the rotational angles, and directions because of the interventions.

Participants

Participants in the study were 17 preservice teachers enrolled in two sections of a prerequisite mathematics content course, taken one semester prior to entering their intended education majors – either early childhood (birth to 2^{nd} grade) or elementary education (Kindergarten to 6^{th} grade) – at a state university in the United States during the Spring 2025 semester. All participants were females, with 6 enrolled in the section that received the digital tool intervention and the remaining 11 in the section that received the physical tool intervention. A few students from the two sections were excluded from this study due to their absence during the intervention sessions. Table 1 presents the distribution of participants' majors in terms of frequency count and percentage.

Table 1. Participant Major Distribution (N = 17)

Intervention Group	Major	Frequency	%
GeoGebra (N = 6)	EE	4	66.7%
	EL	2	33.3%
Tracing-paper (N = 11)	EE	8	72.7%
	EL	3	27.3%

Note. EE = Early Childhood major, EL = Elementary Education major.

Interventions

The interventions were conducted separately by two experienced university instructors using a guided exploration approach with around 75 minutes each. One instructor used GeoGebra, a dynamic geometry software, to guide students in constructing and rotating figures on a digital Cartesian coordinate system while the other instructor used tracing paper to direct students physically manipulate and observe rotations of geometric figures around different centers of rotation.

The Use of GeoGebra (Digital Tool)

For this intervention group, the instructor found that all six participants had little experience with GeoGebra before the lesson of rotation. Therefore, she spent the first 30 minutes introducing its fundamental features, including navigating each drop-down menu; constructing points, line segments, and polygons; measuring lengths and angles; moving geometric figures on the coordinate plane; and using the undo function. The initial instruction was essential to get students familiar with the digital tool and reduce technical confusion. In the subsequent section of the lesson, the instructor guided them in constructing triangle ΔABC (the pre-image) on the coordinate plane and applying the rotation function to perform a 90º clockwise rotation about the origin (D) to form the image ΔA'B'C' (see an example in Figure 1). Students were then encouraged to create pre-images with varied vertex coordinates and internal angles. Once they felt comfortable, they were free to adjust rotation parameters, construct various triangles, and observe the effects in real time.

The instructor divided students into two groups to discuss questions related to the changes of side lengths and angles, such as: "What happens to the distances from the corresponding vertices to the center of rotation (D), such as AD and A'D, BD and B'D, and CD and C'D?" "What happens to the angles ∠ADA', ∠BDB', and ∠CDC'? Are they the same or different? Why?" and "What do you need to know to draw a rotation?" The lesson was concluded with a whole-class discussion, during which each group shared their observations to these questions.

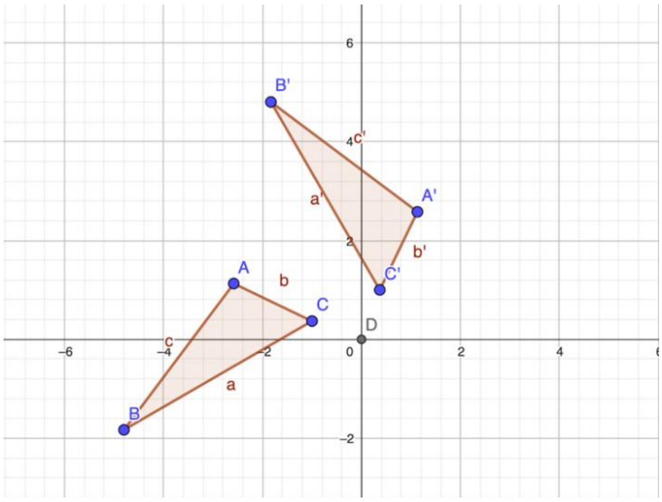


Figure 1. A 90º Clockwise Rotation of ΔABC to ΔA'B'C' about the Origin (D)

The Use of Tracing Paper (Physical Tool)

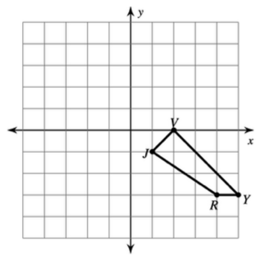
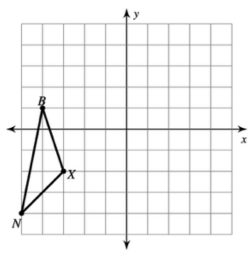
For this intervention group, the instructor spent the first 40 minutes demonstrating the use of tracing paper for drawing rotations and providing students with opportunities to practice. She began by drawing a point A (the pre-image) and another point O as the center of rotation on the whiteboard. She then placed a tracing paper over the pre-image, traced and labeled point A and the center of rotation O, and, while holding the pen at the center, rotated the paper 90º clockwise about the origin to identify the image A'. She gave students a similar task, asking them to individually draw a 90º counterclockwise image. The instructor circulated around the classroom to help address difficulties, such as identifying the coordinates of a point. She continued by demonstrating and guiding students through a series of increasingly challenging tasks, including rotations of 180º and 270º, using centers of rotation other than the origin, and using various pre-images such as a line segment, triangle, quadrilateral, and regular polygon.

For the remaining 35 minutes, the instructor showed three short videos (2-3 minutes each) to review how to perform rotations using tracing paper and encouraged students to describe the procedure in their own words. Afterward, students worked in pairs on rotation tasks and shared their solutions with the whole class at the end.

Assessment Instrument

A content knowledge assessment was administrated to the participants before and after the implementation of the interventions. The assessment consists of five questions based on key components of the rotation concept that middle school students are expected to understand in order to describe the effects of rotations on two-dimensional figures using coordinates (NGA Center & CCSSO, 2010). The first three questions are multiple-choice, and the last two require drawing a rotated image with all vertices counted toward the percentage score. Due to the small number of items and the fact that each question was designed to assess one distinct conceptual component individually, Cronbach's alpha was not calculated, as it may underestimate the internal consistency in such cases. However, content validity was supported through expert review and revision by two mathematics specialists who have extensive experience teaching mathematics at the middle and high school levels and are familiar with the Common Core geometry standards. Question items are presented in Table 2.

Table 2. Question Items Used in The Content Knowledge Assessment

Key Content Components	Question
Rotating about the origin	A triangle with one vertex at A (3,2) is rotated **90° counterclockwise about the origin**. What are the coordinates of the image of point A? **A.** (2, 3) **B.** (–2, 3) **C.** (–3, 2) **D.** (–2, –3)
Rotating about an arbitrary center	A point P (4,1) is rotated **180° clockwise about the point (2, 2)**. What are the coordinates of the image of point P? **A.** (0, 3) **B.** (0, 1) **C.** (0, 2) **D.** (1, 0)
Angle of rotation	A regular hexagon is rotated about its center. What is the minimum **angle required for rotational symmetry**? **A.** 60° **B.** 90° **C.** 120° **D.** 180°
Graphing a rotated image about the origin	Graph the image of the figure after a 180° clockwise rotation about the origin 
Graphing a rotated image around a point other the origin	Graph the image of the figure after a 90° counterclockwise rotation about the origin 

Data Analysis

This study used SPSS (Version 29.0.2.0). Descriptive data were analyzed to assess preservice teachers' baseline content knowledge prior to the interventions, which focused on the centers of rotation, rotational angles, and directions. A minimum mastery threshold of 60% correctness for each question and for overall performance was adopted (Venkat & Spaull, 2015), as the content knowledge is typically taught in school as part of preservice teachers' prior knowledge. Due to the small sample size ($N < 30$), the Wilcoxon signed-rank test was employed to examine the differences in overall content knowledge before and after the interventions. The Wilcoxon test is a robust statistical tool for small sample size or non-normally distributed data. It is widely used in medical research to compare treatment effectiveness (FasterCapital, 2025) and is therefore appropriate for detecting the intervention effectiveness in this study. Similarly, another nonparametric statistical test – the Mann-Whitney U test – was used to compare knowledge gains over time between the GeoGebra and Tracing-paper intervention groups.

Results

Research Question 1: What Foundational Content Knowledge Do Preservice Teachers Possess About Rotation and Its Key Components?

Before the interventions, preservice teachers' baseline content knowledge was assessed, and the descriptive statistics are presented in Table 3. Each correct answer received 1 point, and the correct percentage indicates the proportion of preservice teachers who answered each question correctly. For Question 1 (Identify the image of A (3, 2) after a 90° counterclockwise rotation about the origin), 3 out 17 participants selected the correct answer, resulting in a mean score 0.18 ($SD = 0.39$), indicating 17.6% of preservice teachers answered correctly, with moderate variability in responses. For Question 2 (Identify the image of P (4, 1) after 180° clockwise rotation about the point (2, 2)), 6 participants answered correctly, yielding a mean score 0.35 ($SD = 0.49$), suggesting 35.3% preservice teachers answered correctly, with moderate variability. For Question 3 (Identify the minimum angle for a rotational symmetry of a hexagon), 3 participants selected the correct answer, with a mean score 0.18 ($SD = 0.39$), indicating 17.6% of preservice teachers answered correctly, with moderate variability.

Table 3. Preservice Teachers' Baseline Content Knowledge on Rotation

Question	M	SD	Correct Frequency	Correct Percentage (%)
Rotating 90° about the origin	0.18	0.39	3	17.6%
Rotating 180° about non-origin point	0.35	0.49	6	35.3%
Angle of rotation	0.18	0.39	3	17.6%
Graphing a 180° rotation	0.25	0.39	3.75	22.0%
Graphing a 90° rotation	0	0	0	0%
Pretest total (Maximum = 5pts)	0.96	0.86	3.15/item	18.5%

Notes. Each item is scored 0 (incorrect) or 1 (correct). Correct% = (Total correct frequency)/Total number x 100%.

Questions 4 and 5 require drawing a graph after a 90° or 180° rotation about the origin, with all vertices counted toward the percentage score. The result indicates that 22% of participants were able to graph a quadrilateral rotated 180° clockwise about the origin, while no preservice teacher could correctly locate even one vertex of the image for a 90° rotation.

Based on the results displayed in Table 4, the average proportion of participating preservice teachers who answered the questions correctly was 18.5%, which is well below the expected content mastery standard of 60% (Venkat & Spaull, 2015). The lowest correct response rates were observed on tasks of identifying or graphing an image after a 90° rotation (Q1 = 17.6%, Q5 = 0%), while the relatively highest rates were for tasks involving a 180° rotation (Q2 = 35.3%, Q4 = 22.0%). The question assessing knowledge of rotational angles also received a low correct response rate of 17.6%. Only 1 out 17 preservice teachers answered 3 out 5 (60%) questions correctly, meeting the proficiency threshold level prior to the interventions.

Research Question 2: Does the Use of Digital or Physical Tools Improve Preservice Teachers' Understanding of Rotation, and If so, in Which Aspects?

Because of the small sample size (N = 17), we used a Wilcoxon signed-rank test to compare participants' content knowledge before and after the interventions (see Table 4). The results showed a significant increase in scores, Z = 3.53, $p < 0.001$. Using the formula $r = Z / \sqrt{N}$, the effect size was calculated as $r \approx 0.86$, indicating a large effect. In other words, the use of either digital or physical tools had a significant impact on preservice teachers' understanding of rotations.

Table 4. A Wilcoxon Signed-Rank Test on Pre- and Post-Test Scores

Measure	Pre-Median	Post-Median	Z	P	Effect Size (r)
Total Accurate responses	1.00	3.75	3.53	< 0.001	0.86

Note. The maximum possible total of accurate responses is 5, based on the 5 question items.

Based on the collected data, 16 out of 17 participants showed improvement in their post-test scores, while 1 participant scored lower. In addition, 11 participants answered 3 or more out of 5 questions correctly, indicating that 64.7% met or exceeded the proficiency threshold of 60% correctness after the interventions. An average mean gain of 2.41 indicates the participants answered 2.41 more items correctly on the post-test. Surprisingly, the most challenging questions with 90° rotations showed the highest gains (Q1 with 58.9%, Q5 with 52.9%). Item-wise, 4 out 5 items were answered correctly by more than 60% of preservice teachers, while only the question involving graphing a 90° rotation received a slightly lower correct rate of 52.9% (see Table 5).

Additionally, a between-group comparison was conducted with means and standard deviations (see Table 6). The group using GeoGebra had a mean score of 1.00 (SD = 0.89) for accurate responses on the pretest and 3.17 (SD = 1.13) on the posttest, indicating that, on average, participants correctly answered 3.17 out of 5 items, with a gain of 2.17 items after the intervention. Likewise, the tracing-paper group showed a substantial improvement from the pretest mean of 0.93 (SD = 0.89) to the posttest mean of 3.48 (SD = 1.67), reflecting an increase of 2.55 items (see Figure 2). The results suggest that both digital and physical tools positively impacted preservice teachers' content knowledge on rotations.

Table 5. Pre- and Post-Test Scores on Content Knowledge of Rotation

Question	Pre M	Pre-SD	Pre-Correct%	Post M	Post SD	Post-Correct%	Gain%
Rotating 90° about the origin	0.18	0.39	17.6%	0.76	0.44	76.5%	58.9%
Rotating 180° about a non-origin point	0.35	0.49	35.3%	0.65	0.49	64.7%	29.4%
Angle of rotation	0.18	0.39	17.6%	0.65	0.49	64.7%	47.1%
Graphing a 180° rotation	0.25	0.39	22.0%	0.76	0.37	64.7%	42.7%
Graphing a 90° rotation	0	0	0	0.55	0.50	52.9%	52.9%
Total (Max = 5pts)	0.96	0.86	18.5%	3.37	1.47	64.7%	46.2%

Notes. Each item is scored 0 (incorrect) or 1 (correct). Correct% = (Total correct frequency)/Total number × 100%.

Table 6. Pre- and Post-Test Scores for Both Intervention Groups

Intervention Group	Pre-Mean	Pre SD	Post Mean	Post SD	Mean Increase
GeoGebra Group (N = 6)	1.00	0.89	3.17	1.13	2.17
Tracing Paper Group (N = 11)	0.93	0.89	3.48	1.67	2.55

Notes. Each item is scored 0 (incorrect) or 1 (correct). The maximum possible mean score is 5, based on 5 items.

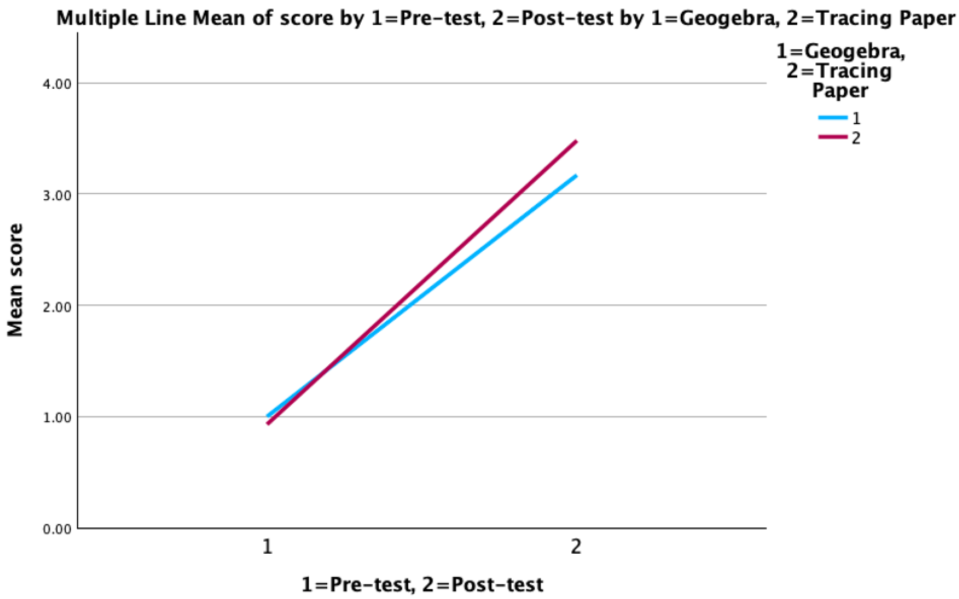


Figure 2. Line Plot of Pre-Posttest Mean Scores for Two Intervention Groups

It was interesting to observe that both experimental groups showed substantial gains over the interventions, with a slightly stronger effect for the Tracing Paper Group. Therefore, an Independent-Samples Mann-Whitney U test was conducted to further examine whether a significant difference exists between two instructional tools in preservice teachers' understanding of rotations. The results showed no statistically significant difference between the two groups, $U = 40.50$, $z = 0.76$, $p = 0.462$, which is above the significance level of 0.05. Hence, we failed to reject the null hypothesis, which states that there is no difference in student knowledge gains between the two interventions (see Table 7).

Table 7. Independent-Samples Mann-Whitney U Test Summary

Null Hypothesis	**Test**	**Sig.**^{a,b}	**Decision**
The distribution of Gains is the same across groups of 1 = GeoGebra, 2= Tracing Paper	Independent-Samples Mann-Whitney U Test	0.462^{c}	Retain the null hypothesis.

Notes. A. The significance level is .50. B. Asymptotic significance is displayed. C. Exact significance is displayed for the test.

Research Question 3: How Do the Two Interventions Differ in Their Effects on Key Aspects of Preservice Teachers' Understanding of Rotation?

Based on the posttest data, 3 out of 6 participants (50%) in the GeoGebra Group versus 8 out of 11 participants (72.7%) in the Tracing Paper Group met or exceeded the proficiency threshold of 60% correctness on the content test. As shown in Table 8, the largest gains for the GeoGebra Group occurred in understanding 180° rotations (Q2 and Q4) and identifying the angle of a rotation (Q3), while the lowest gains were observed in 90° rotations (Q1 and Q5). In contrast, for the Tracing Paper group, the largest gains are in identifying or graphing 90° rotations (Q1 and Q5), with the lowest gains in identifying a rotation about a non-origin point (Q2) and determining the angle of rotation (Q3). These findings suggest a complementary effect, even though both interventions resulted in substantial gains in preservice teachers' content knowledge related to rotation. Additionally, it is notable that the group using a physical tool (72.7%) outperformed the group using a digital tool (16.7%) in graphing a 90° rotation, despite both groups starting with minimal prior knowledge (0%).

Table 8. Pre- and Post-Test Mastery Percentages for Both Intervention Groups

Question	**Pre% (G1)**	**Post% (G1)**	**Gain% (G1)**	**Pre% (G2)**	**Post% (G2)**	**Gain% (G2)**
Rotating 90° about the origin	33.3%	66.7%	33.4%	9.1%	81.8%	72.7%
Rotating 180° about non-origin	33.3%	83.3%	50%	36.4%	54.6%	18.2%
Angle of rotation	16.7%	83.3%	66.6%	18.2%	54.6%	36.4%
Graphing a 180° rotation	16.7%	66.7%	50%	29.6%	81.8%	52.2%
Graphing a 90° rotation	0%	16.7%	16.7%	0%	75.7%	75.7%
Overall	20%	63.3%	43.3%	18.7%	69.7%	51.0%

Note. G1 represents the GeoGebra Group, and G2 represents the Tracing Paper Group.

In conclusion, although both digital and physical tools positively impact preservice teachers' understanding of rotation, their effects were disproportionately distributed across varied yet complementary aspects. Therefore, it is possible to integrate both tools into instructions to achieve the desired learning outcomes.

Discussion and Conclusion

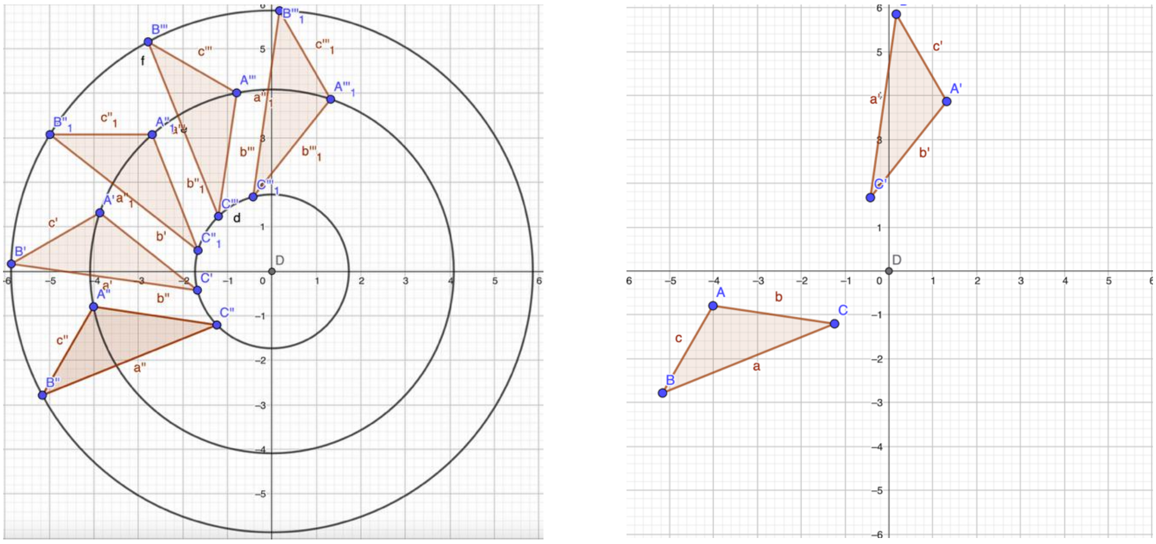
The novelty of this study lies in its focus on the under-investigated topic of preservice teachers' understanding of rotation, as well as the impact of interventions with instructional tools on their knowledge gains. Accordingly, the effectiveness of a digital tool (GeoGebra) and a physical tool (Tracing Paper) was examined and compared, thereby addressing a missing component in the research literature.

In the present study, only 1 out of 17 (6%) participants reached the 60% proficiency level prior to the interventions, and most struggled with identifying and drawing rotated images as well as determining rotation angles. The results revealed that most preservice possessed a limited understanding of rotation and its key components. These findings are consistent with previous research in geometry (Mbusi, 2021; Salifu et al., 2021), particularly in the area of rotation (Clements & Battista, 1992). Overall, the result highlights the need for stronger content preparation in teacher education programs, supporting Ball et al.'s (2008) emphasis on the essential role of content knowledge within the framework of teacher knowledge for teaching.

Furthermore, this study investigated the effectiveness of both digital and physical tools in fostering preservice teachers' understanding of rotation through a quasi-experimental, pretest-posttest design with non-equivalent groups (Reichardt, 2009). A 75-minute intervention using either GeoGebra or tracing paper was delivered separately to groups of preservice teachers. Based on the results of a Wilcoxon Signed-Rank test, most participants' content knowledge improved significantly over time, with 11 out of 17 (65%) meeting or exceeding the 60% proficiency threshold. These findings provide further evidence of the value of both digital and physical tools in math education (Carbonneau et al., 2013; Hoyles & Noss, 2003; Kaput, 1992; Moyer-Packenham & Westenskow, 2013). Interestingly, no significant differences were identified between the intervention groups, and somewhat unexpectedly, the Tracing Paper Group scored slightly higher than the GeoGebra Group. This result contradicts earlier findings by Moyer-Packenham and Westenskow (2013), who reported that virtual manipulatives were more effective.

In addition, varying rates of knowledge gains for key components of rotation were found in the two intervention groups, suggesting that digital and physical tools could complement each other for teaching this topic. The largest gains for the GeoGebra Group occurred in understanding 180° rotations and identifying the angle of a rotation, whereas the Tracing Paper Group showed the largest gains in identifying vertices for and graphing 90° rotations. These differences could be attributed to the processes of using each tool and corresponding cognitive development they facilitate. On one hand, using tracing paper allows learners to manually rotate a figure and visualize the entire movement (as shown in the left graph of Figure 3). This dynamic process may facilitate a conceptual understanding that each vertex and its images are equidistant from the center (forming a radius) and that the rotational angles are congruent for all vertices from the original figure to its image. On the other hand, GeoGebra illustrates the original figure and its image without showing the motion. This digital tool functions similarly to a calculator, where the input sequence is "Click on the figure and the center, then rotate clockwise 120º" to produce an output image (see the right graph of Figure 3).

Therefore, a promising instructional approach would be to integrate both tools: beginning with the use of tracing paper to develop tactile experiences that allow learners to visualize the entire rotation process and form a conceptual understanding of the positions of all vertices and sides before and after a rotation. This could then be followed by using GeoGebra to complete rotations with more complicated angles for reinforcement and deeper exploration of the relationships among sides and angles.



Rotation process with tracing paper Rotation process with Geogebra

Figure 3. Examples of A Triangle Rotated Clockwise 120º Using Both Tools

Findings from this study reveal that both digital and physical tools are useful in promoting preservice teachers' understanding of the concept of rotation. Instructors may consider integrating both tools for challenging geometric topics, either within teacher preparation programs or in the K-12 classrooms.

Another consideration arises from the methodological limitations of this study. Future studies could consider incorporating qualitative data collection to explore what challenges or benefits that preservice teachers experience when using GeoGebra and tracing paper. In addition, more teacher participants should be recruited for the experiments to help increase the generalizability of the findings.

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